

### 3. BOUNDARY LAYER CONCEPTS AND

#### CLOSED CONDUCT FLOW

$$\left(\frac{v_b}{U}\right) \mu = \tau_{\text{c}} \text{ where } \mu = \text{constant}$$

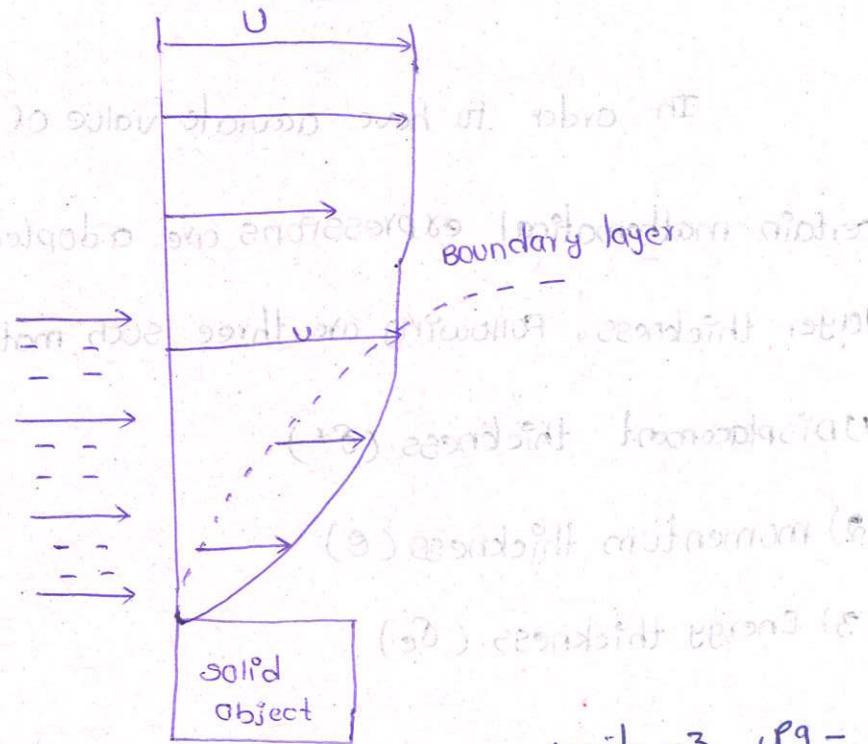
#### Boundary layer concept:

Boundary layer is a narrow region in the immediate vicinity of solid object in which the fluid have variation of velocity from zero to the velocity of main stream asymptotically.

Theory which explain this concept is called as boundary layer concept.

whenever a real fluid comes across a solid object during its motion. it adheres to the solid, because of viscosity property. At this moment, the relative velocity between fluid and solid object is zero.

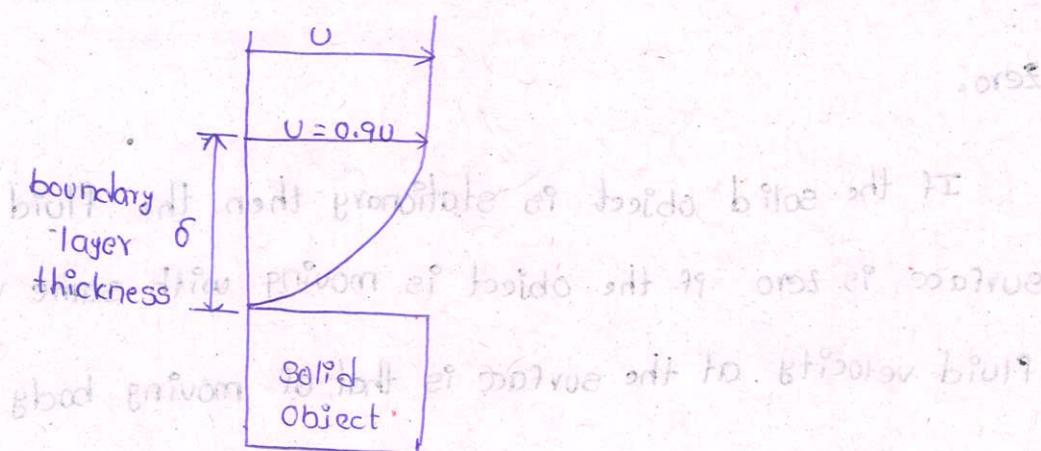
If the solid object is stationary then the fluid velocity at the surface is zero if the object is moving with same velocity then the fluid velocity at the surface is that of moving body



In boundary layer region, velocity gradient,  $\frac{dv}{dy}$  normal to the solid surface is very high since there is large variation of velocity in a small distance, shear stress  $T = \mu \left( \frac{dv}{dy} \right)$ .

### Thickness of boundary layer:

Boundary layer is a small region in the vicinity of solid body where the velocity of the real fluid changes from zero to main stream velocity asymptotically. The boundary layer thickness is defined as the distance from the solid body surface to the point where the velocity of liquid reaches 99% of main stream velocity ( $U = 88\% U$ ). Hence in general it is termed as nominal thickness of boundary layer. It is denoted by  $\delta$ .



In order to have accurate value of boundary layer thickness certain mathematical expressions are adopted to measure the boundary layer thickness. Following are three such mathematical expression

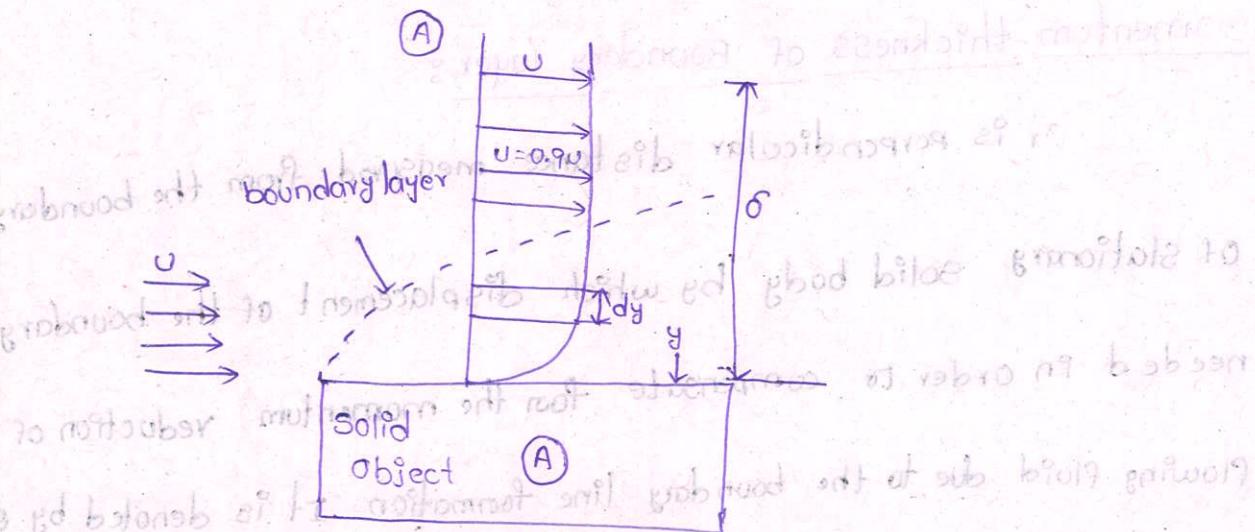
1) Displacement thickness ( $\delta^*$ )

2) momentum thickness ( $\theta$ )

3) Energy thickness ( $\delta_e$ )

## Displacement thickness:

In order to compensate flow rate reduction which occurs due to the boundary layer formation, an additional wall thickness is to be added to the boundary surface. This additional wall thickness is termed as displacement thickness.



It is also defined as perpendicular distance measured from the boundary by which displacement of free stream took place due to boundary layer information.

It is a mathematical expression, which is used to provide accurate effect of boundary layer thickness on flow of liquid if it is denoted by  $\delta^*$

$$\text{mass of fluid through the elementary } = \rho \times \text{velocity} \times \text{area} \\ = \rho U dy$$

$$\text{mass of fluid through secondary element} = \rho U dy$$

$$= \rho U (\bar{U} - U) dy$$

$$\text{total reduction of mass flow } = \int_0^\infty \rho (U - \bar{U}) dy \quad \text{--- (2)}$$

$$\text{loss of fluid flowing through distance } \delta^* \text{ is } \rho U \delta^* \quad \text{--- (3)}$$

from equation (2) and (3)

$$= \int_0^{\delta} \rho (U - u) dy = \rho U \delta^*$$

constant flow  $\frac{\int_0^{\delta} \rho (U - u) dy}{\rho U} = \delta^*$  speed gradually out of sub

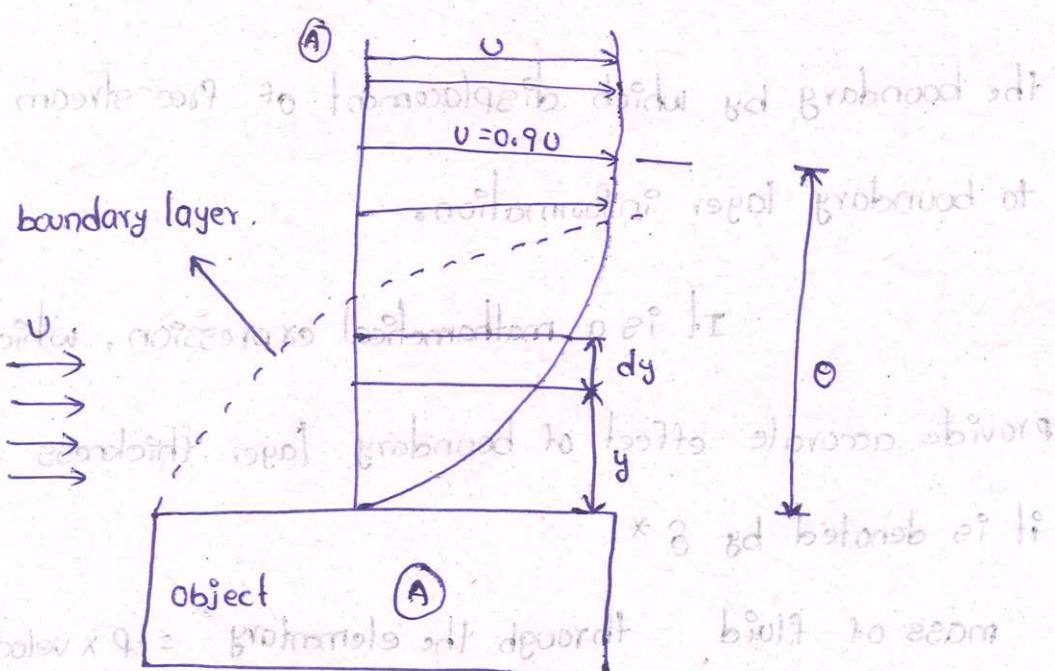
$$\therefore \delta^* = \int_0^{\delta} \left[ 1 - \frac{u}{U} \right] dy$$

### Momentum thickness of Boundary Layer

It is perpendicular distance measured from the boundary

of stationary solid body by which displacement of the boundary is needed in order to compensate for the momentum reduction of the flowing fluid due to the boundary line formation. It is denoted by  $\theta$ .

It is used in kinetics



### Momentum thickness of boundary layer

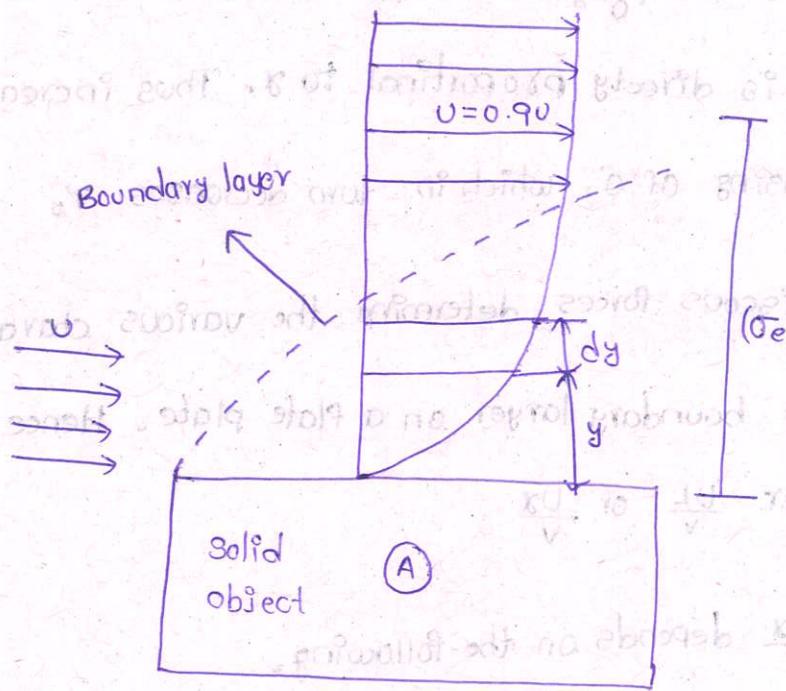
$$\int_0^{\delta} \rho U (U - u) dy = \rho \theta U^2$$

$$\therefore \theta = \int_0^{\delta} \frac{\rho U}{\rho U^2} (U - u) dy = \frac{1}{U} \int_0^{\delta} (U - u) dy$$

## Energy thickness:

It is defined as perpendicular distance measured from the boundary of the stationary solid body by which displacement of boundary is needed in order to compensate for kinetic energy reduction of the following fluid due to boundary layer formation.

→ it is denoted by ( $\delta_e$ )



Energy thickness boundary layer

Energy thickness ( $\delta_e$ )

$$\int_0^{\delta_e} \frac{1}{2} \rho U (U^2 - u^2) dy = \frac{1}{2} \rho \delta_e U^3$$

$$\int_0^{\delta_e} \frac{u(U^2 - u^2)}{U^3} dy = \delta_e$$

$$\therefore \delta_e = \int_0^{\delta_e} \frac{u}{U} \left[ 1 - \frac{u^2}{U^2} \right] dy$$

## \* characteristics of boundary layer:

- Thickness of boundary layer decreases if the value of velocity of main stream  $U$  increases.
- The thickness of boundary layer increases with increase in kinematic viscosity ( $\nu$ )
- Thickness of boundary layer increases with the distance from leading edge  $x$  increases
- Shear stress  $\tau_0 \approx \mu \left[ \frac{U}{\delta} \right]$  hence  $\delta'$  decreases with the increasing of  $x$ , since  $\delta'$  is directly proportional to  $x$ . Thus increasing in  $x$  result in increasing of  $\delta'$ , which in turn decreases  $\tau_0$ .
- Inertial and viscous forces determine the various characteristics like  $\delta$ ,  $\delta'$ ,  $F$  of the boundary layer on a flat plate. Hence they are the function of either  $\frac{UL}{\nu}$  or  $\frac{Ux}{\nu}$
- The values of  $\frac{Ux}{\nu}$  depends on the following.
  - (i) pressure gradient
  - (ii) temperature difference between fluid and boundary
  - (iii) turbulent in ambient flow
  - (iv) roughness of surface
  - (v) curvature of plate
- Velocity distribution is a parabolic i.e. boundary layer is laminar if  $\frac{Ux}{\nu}$  is less than  $5 \times 10^5$
- Velocity distribution follows power law i.e. boundary layer is turbulent if  $\frac{Ux}{\nu}$  is greater than  $5 \times 10^5$

problem

The boundary layer thickness at 4 m from the leading edge of a flat plate kept over zero angle of to the flow direction is 2mm. The free stream velocity is 25 m/s calculate the boundary layer thickness at 8 m.

Given that

$$\text{Distance } x_1 = 4 \text{ m}$$

$$\text{Distance } x_2 = 8 \text{ m}$$

Boundary layer thickness  $\delta_1 = 2 \text{ mm}$

free stream velocity  $U = 25 \text{ m/s}$

Boundary layer thickness  $\delta_2 = ?$

Thickness of boundary layer is given by

$$\delta = \frac{5x}{\sqrt{Re_x}}$$

$$\delta = \frac{5x}{\sqrt{\frac{Ux}{V}}}$$

$$\delta = 5 \sqrt{\frac{Ux}{V}}$$

At distance  $x_1$ , thickness of boundary layer is

$$\delta_1 = 5 \sqrt{\frac{Ux_1}{V}}$$

at distance  $x_2$ , thickness of boundary layer is

$$\delta_2 = 5 \sqrt{\frac{Ux_2}{V}}$$

Dividing equations ① and ②

$$\frac{\delta_1}{\delta_2} = \frac{5 \sqrt{\frac{Ux_1}{V}}}{5 \sqrt{\frac{Ux_2}{V}}}$$

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}}$$

substituting the values of  $\delta_1$ ,  $x_1$ , and  $x_2$  in above equation

$$\frac{x}{\delta_2} = \sqrt{\frac{4}{8}}$$

$$\frac{x}{\delta_2} = \sqrt{\frac{1}{2}}$$

$$\delta_2 = 2.829 \text{ mm}$$

Boundary layer thickness at a distance of 8m is 2.829 mm.

### Laminar boundary layers

- In laminar boundary layer every layer slides over the adjacent layer
- The exchange of mass or momentum takes place only between adjacent layers over a microscopic scale.
- These layers are formed only when the Reynolds numbers are small
- The skin friction for laminar flow is given as

$$C_f = \frac{0.664}{\sqrt{Re_x}}$$

where  $Re_x$  - Reynolds number based on the length of plate.

$$\frac{Re_x}{U} = \frac{L}{\nu}$$

In turbulent boundary layer there is mixing of several layers over the microscopic scale.

The exchange of mass, momentum, and energy in turbulent boundary layer is on a greater scale, when compared to laminar boundary layer.

These layers are formed only when the Reynolds numbers are large.

The skin friction for turbulent flow is given as

$$C_f = \frac{0.0594}{Re^{0.2}}$$

where

Re - Reynolds number based on the length of plate.

### Separation of an arbitrary boundary layer:

separation of boundary layer from surface of solid body

may take place if these forces act as over a long distance. Thus

resulting in the shift of boundary layer into main stream. This

phenomenon is called as separation of boundary layer and the

point of separation is that point on the body at which the boundary

layer is on verge of separation from the surface.

As the fluid flows around the curved surface from

point 1 to point 2.

The area of flow decreases and hence velocity of flow increases.  
due to increase of velocity pressure decreases in the flow direction  
and hence the pressure gradient is negative i.e.  $\frac{dp}{dx} < 0$ . Boundary  
layer moves forward till pressure gradient is negative  
when fluid pass point  $a$  beyond it the pressure gradient is  
positive since pressure increases due to the decrease of velocity  
and due to increase of area of flow.

### Different methods of controlling the separation of boundary layer

Boundary layer separation is phenomenon at which the boundary layer  
will be separate from the solid body surface.

Due to this continuous loss of energy takes place which is undesirable  
feature, thus methods must be adopted in order to overcome this  
problem.

Various methods that are used for controlling the separation of boundary  
layer are as follows.

- 1) Streamline of boundary shapes
- 2) Acceleration of fluid in the boundary layer
- 3) Motion of solid boundary
- 4) Suction of fluid from the boundary layer.

## stream line of body shapes:

The skin friction drag can be reduced by using suitably shaped bodies since the boundary layer's points of transition from laminar to turbulent can be moved downstream

- 2) Acceleration of fluid in the boundary layer:  
In this method, the practice of fluid which are getting related in boundary layer are supplied by additional energy by either dividing a portion of fluid of main stream from high pressure region to retard region by providing slot in the body as shown in the regions.
- 3) motion of solid boundary:  
The main and the only cause for formation of boundary layer is because of the velocity difference between the solid boundary and flowing fluid. This can be eliminated by rotating emerged in the fluid.

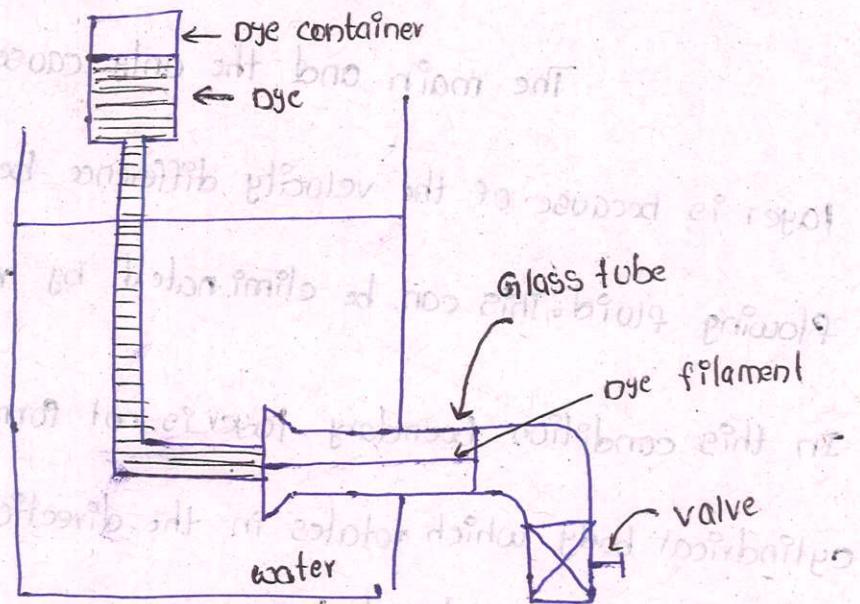
In this condition boundary layer is not formed on the upper side of cylindrical body which rotates in the direction of flow but however the other side of cylindrical body will have separation of boundary layer.

## 4. suction of fluid from the boundary layer:

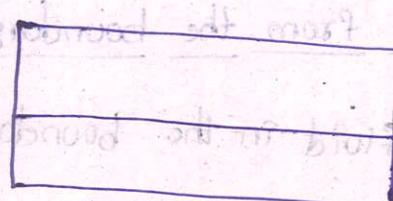
The related fluid in the boundary layer is removed with the help of porous surface or by suction through slot thus eliminating the separation.

## Reynold's Experiment:

Reynold's conducted an experiment to study the fluid flow. i.e laminar and turbulent was first demonstrated by him in this experiment. The experiment consists of a constant head tank containing water a small tank containing dye, a glass tube with a bell mouthed entrance and a regulating valve as shown in fig. Water was allowed to flow from the tank into the atmosphere through the glass tube. Variation in the flow velocity was achieved with the help of a regulating valve. A liquid dye with same specific weight as that of water was introduced into the flow through a small tube at the bell mouthed entrance of the glass tube as shown in fig.



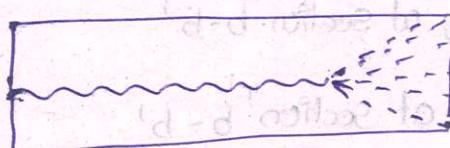
Reynold's apparatus



laminar flow.



transiston flow



turbulent flow

→ it was observed that when the velocity of flow was low, the dye filament when flowing through the glass tube was in the form of a straight line. fluid particles were moving in parallel layers or laminae. This case was laminar flow as shown in figure. when velocity of flow is increased the dye filament no longer remained a straight line but was a wavy one as shown in figure.

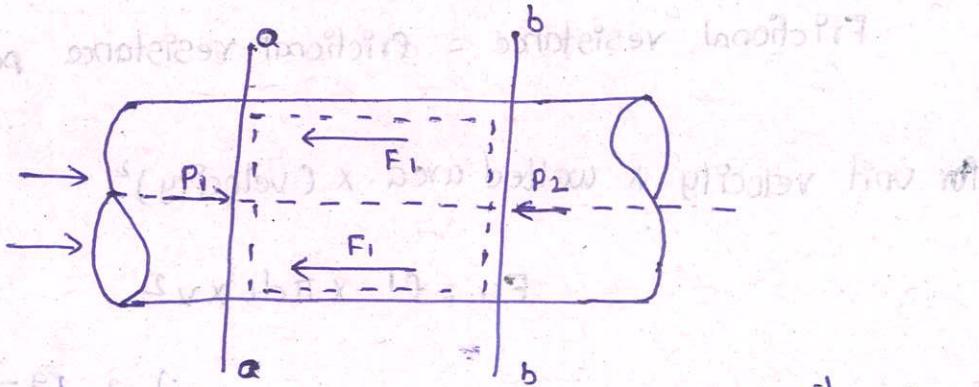
### Darcy's Weisbach equation for turbulent flow:

#### Darcy's Weisbach equation:

Darcy's Weisbach equation is useful for finding the head loss due to friction, when the flow through the pipe is turbulent. The derivation of the equation given below

consider a horizontal pipe with uniform diameter and steady flow

as shown in figure. Let a-a and b-b are two sections of a pipe



let

$p_1$  - pressure intensity at section 'a-a'

$v_1$  - velocity of flow at section 'a-a'

$p_2$  - pressure intensity at section 'b-b'

$v_2$  - velocity of flow at section 'b-b'

$L$  - length of pipe between two sections

$D$  - diameter of the pipe

$f'$  - frictional resistance per unit area per unit velocity

$h_f$  - loss of head due to friction.

By applying bernoulli's equation at sections 'a-a' and 'b-b'

total head at 'a-a' = total head 'b-b' + loss of head due to friction

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_f$$

But

$z_1 = z_2$  as the pipe is horizontal

$v_1 = v_2$  as diameter of pipe is uniform

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f$$

$$h_f = \frac{p_1 - p_2}{\rho g}$$

$$(p_1 - p_2) = \rho g h_f$$

Frictional resistance = frictional resistance per unit wetted area

For unit velocity  $\times$  wetted area  $\times$  (velocity) $^2$

$$F_i = f' \times \pi D L \times v^2$$

$$= f' \times P \times L \times V^2 \quad [ \because \pi D = \text{Perimeter}(P) ]$$

The forces acting on a fluid between sections 'a-a' and 'b-b'

are

1) pressure force at section 'a-a' =  $P_1 A$

2) pressure force at section 'b-b' =  $P_2 A$

3) frictional resistance =  $F_f$

By resolving these forces

$$P_1 A - P_2 A - F_f = 0$$

$$(P_1 - P_2)A = F_f$$

$$(P_1 - P_2) = \frac{f' \times P \times L \times V^2}{A}$$

from equation ①

$$(P_1 - P_2) = \rho g h_f$$

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

$$h_f = \frac{f' \times P \times L \times V^2}{\rho g A}$$

$$h_f = \frac{f'}{\rho g} \times \frac{L}{m} \times V^2$$

where  $m = A/P$  is called hydraulic mean depth or hydraulic radius

$$m = \frac{A}{P} = \frac{\frac{\pi}{4} D^2}{\pi D} = \frac{D}{4}$$

By substituting value of  $m$  we get

$$h_f = \left( \frac{f}{D} \right) \frac{\rho g}{2} \times \frac{4 L V^3}{D}$$

Multiplying and divide the equation by  $\frac{2}{\rho g D}$  and get

$$h_f = \frac{f}{D} \times \frac{4 L V^2}{2 g D}$$

$$\text{let } f = \frac{4 f_1}{D}$$

$$h_f = \frac{4 f_1 L V^2}{2 D g}$$

The above equation is known as barcy-weisbach equation and the term  $f$  is known as coefficient of friction.

It can also be written as

$$h_f = \frac{f_1 L V^2}{D g}$$

where

$$f_1 = 4 f$$